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MINIMUM MATHEMATICAL REQUIREMENTS FOR AGRICULTURAL STUDY

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Twelve years of responsibility for the instruction in mathematics in the College of Agriculture and in some measure in the School of Agriculture, may be found some justification for this presentation with considerable definiteness of certain of my own views on this subject.

General Educational Value of Mathematics. It has never been successfully disputed that in any line of study, mathematics develops certain habits of thinking as does no other line of work. The mastery of mathematics demands the development of the three great elements of good thinking power namely: application, concentration, and power of analysis. The first two of these must necessarily result in: clarity, good logic, breadth of thought, exactness and accuracy.

The power of analysis implies development of the power of: comparison, contrast and the recognition of relative values, all of which are essential in advanced agricultural study.

Mathematics the Basis of Scientific Classification. The science of mathematics is essentially a classified field and so forms the basis for classification in all sciences. That is, in large measure, all science is dependent on mathematics for its development. Agriculture is a specialized field of science, classified according to the various phases of work and each of these phases demands more or less careful mathematical training and ability. These phases might be roughly listed as follows: farm management, dairy production and feeding; surveying, drainage and irrigation; buildings, machinery and power; and the special agricultural sciences, including: plant breeding, pathology and economic zoology; agricultural chemistry and soils; and agricultural economics and rural sociology.

Fundamental Personal Requirements in Mathematics for Agriculture. The fundamental personal requirements in the study of mathematics for agriculture are, in the main, two: first, interest and earnestness on the part of the student in the development of his chosen line of work, and second, the teacher.

The strength of the first of these requirements is very largely dependent upon the second, namely the teacher, who must be possessed, not only of a thorough knowledge of his subject and ability to teach it, but also an unlimited enthusiasm and belief in its practical value. He must be ready and willing to recognize and encourage the individuality of the student, never insisting on teaching by the book or any set method.

A. Special Means and Methods for Slow Students

For students slow to grasp mathematical ideas, carefully drawn charts and mathematical models are often a great help. With any type of student geometrical representation of fundamental algebraic law is excellent and helps to fix such law by means of the avenue of the eye as well as of the mind. The teacher must keep constantly in mind the great end for which he is striving, namely, *understood* method and result, the means to that end being only secondary.

Abolish Artificial Classification and Boundaries. It seems to me especially essential in the teaching of mathematics to students of agriculture that we get away as far as possible from the artificial boundaries and limitations which have been given to the different fields of mathematics. That is, we should not cling to the names: arithmetic, algebra, geometry, etc., but rather speak of *mathematics.* There is no sharp dividing line anywhere in the field. We all recognize that algebraic law is fundamental in mathematics, yet we teach arithmetic first. No one can show where arithmetic leaves off and where algebra begins. We should not attempt to practice it in our teaching. The transition from arithmetic to elementary algebra should be gradual and unnoticeable to the elementary student. By similiar gradual transition, we should pass from algebra to geometry and we should use geometrical illustrations as far as possible all the way. What we need in the way of developed courses are *general* courses in mathematics. A good many of our best teachers of the present day are successfully working on just such a plan.

The Extent of Mathematical Training Needed is Dependent upon the Phase of Agricultural Study to be Followed. It is impossible to place an exact time limit on the study of mathematics for any phase of agriculture study to be pursued by the student. There are four major phases, namely: The Business of Farming; Teaching of Agriculture and Allied Work in Regularly Curricularized Courses; University Extension Work in Agriculture; and Scientific Investigations Relative to Agriculture. Let us consider these in order.

THE BUSINESS OF FARMING

The present day up-to-date farmer must have in his mental equipment sufficient, well-understood mathematics to enable him to carry out mathematical operations which may come into his work of conducting the farm intelligently and profitably. He should be able to measure up and compute the areas of his various fields and to understand the geometrical methods for straightening their boundaries. If he wishes to paint his barn, he should have the knowledge at hand that would enable him to compute closely the amount of paint he will need for the purpose. He should have a knowledge of the principles of percentage as applied to investments, depreciation, income, loans and capitalization. For example, if he is contemplating an enlargement of his business or going into some new field, he must know something of the value of his time and understanding. Assuming that in his general farming operations, he has made a clear profit of $2,400 for a given year, if money be worth 6%, he should be able to understand that his time should capitalize at $40,000. He should be able to draw simple plans, both of his farmstead and his simpler buildings and he should be able to make up comparative tables or charts, through a series of years, of yields and returns therefrom of different types of crops; and so we might go on. The avenue in our educational system which provides for the business of farming is the agricultural high schools and our schools of agriculture. The recognized entrance requirements for these are the completion of the work of our regularly established eighth grade, calling for a complete course in arithmetic, the high schools and agricultural schools following this up with special advanced courses in what is known as farm arithmetic. I have no patience at all with the idea of requiring long and arduous labor by the farm boy or girl in mastering the intricacies of bank discount, annuities, stocks and bonds, alligation, and compound proportion. For such a child, looking forward to work in the School of Agriculture, these topics may well be largely omitted and the time given instead with profit to the following.

(a) Extensive practice in the fudamental operations with the numerous and easy short cuts, such as: to multiply by 125, annex three ciphers and divide by 8; to divide by 25, point off two places and multiply by 4; to lay off a right angle form a triangle with 3 lines in the ratio of 3, 4 and 5.

(b) More work in common fractions and the various processes of reduction of same.

(c) The common operations of percentage as applied in problems of simple interest, investments, income, and simple accounting.

(d) The formulas, with applied problems, of elementary mensuration, including: the areas of all simple plane figures such as the triangle, rectangle, parallelogram, circle, sector, segment, and brief rules for short cuts where these are readily available. Illustrations are: area of circle equal 3 1-7 times its diameter; Area of a segment equals its height times the length of the chord, times the sum of $\frac{2}{3}$ and $\frac{1}{2}$ the square of the ratio of the height to the length of the chord; that is, $A = al(\frac{2}{3} + \frac{1}{2}[a/l]^2)$ in which A stands for area; a, the altitude or middle ordinate of the segment and l the length of the chord. (From Eng. News-Record, Vol. 79, Nov. 22, 1917.)

(e) Volumes of simple solids as the cube, prism, pyramid, cone, frustrum of cone or pyramid and the sphere.

(f) The general nature of the right-angled triangle, including square and cube root.

(g) Most important of all, *rigorous and varied training in problem analysis.*

There is no other part of the average student's preparation in mathematics of which we hear more complaint from their instructors in other subjects than of their inability to handle common fractions and decimals. It is pitiful, too, when an instructor in carpentry gives the dimensions and shape of a building to find that an otherwise capable student is utterly unable to compute the paint required to paint the building only because he does not know how to figure the area of a triangle or is unable to compute the shingles required for the roof because he does not understand the simple principle of mensuration of plane figures. There is something radically wrong either with the instructor or the mental growth of the boy who, as a freshman in college, startles his instructor with the question: "Mr.——, what is the formula for the area of a rectangle? I can't remember it?"

Elementary algebra is usually omitted from eighth grade work and from high school and agricultural school entrance requirements. I believe that the elementary algebraic concepts may well be introduced in the eighth grade work in arithmetic. I have found the algebraic conception of the simple equation and the use of symbols for the unknown quantities a great help to my own children taking problem analysis in eighth grade work. I do not believe in teaching demonstrative geometry in the eighth grade nor in fact without being preceded by considerable instruction in elementary algebra but I do thoroughly believe in extensive training in those phases of arithmetical mensuration drawn from the geometrical laws for areas and volumes. At no other point in a pupil's preparation have I found so pitiful a weakness in both school and college as in the average ability to analyze the thought in a worded problem. I am aware that this matter often resolves itself into a question of syntax, of understanding of English grammar, if you please, but even so it is a fundamental need in mathematics. If the eighth grade teacher will take to heart the suggestions here offered and lay special stress on the particular topics above mentioned, he will very greatly aid the students in our agricultural high schools and schools of agriculture in mastering the necessary more advanced work in elementary mathematics. Yet some study in both elementary algebra and plane geometry is, to my mind, essential even for the elementary student in agriculture. In presenting them, however, the courses as usually given in our text books should be very carefully and fully abridged to fit the time and the need of the student so that his efforts shall not be wasted on protracted discussions and theoretical methods in lowest common multiple, highest common factor, square and cube root and other specialized topics having little useful part in the work. He should rather give his time to the mastering of the essential concepts of algebra, strengthened by a rigorous course in problem analysis. Such a course will equip him not only for his geometry but for any other phase of mathematical work in elementary science that will naturally come into his course of study.

TEACHING OF AGRICULTURE AND AGRICULTURAL EXTENSION

The avenue of our educational system leading to preparation for this work of teaching and agricultural extension is the College of Agriculture of the University. The preparatory school requirements for entrance to the College of Agriculture are very similiar to those for entrance to any other department of the University, mathematics not excepted; namely, a complete regular high school course of four years, including at least one year of elementary algebra through the simpler types of quadratic equations, and one year on the complete course in plane geometry as ordinarily presented in our standard texts. Students looking forward to entering College of Agriculture are further advised to take a third year of mathematics in their high school work covering half a year of advanced work in elementary algebra and half a year in solid geometry. The limitations discussed for School of Agriculture students in algebra and geometry should not in general apply to regular high school students preparing themselves for work in the College of Agriculture. Such students should in general cover the entire course in elementary algebra and in plane geometry with great thoroughness. Those students who come to the College of Agriculture with only two years of high school mathematics are required to take a special course in applied elementary mathematics intended to cover their special needs in mathematical work in undergraduate study. This course extends through a quarter of twelve weeks, five recitations per week and is the practical equivalent in effort and subject matter of a half year's work in the high school. It takes up a few of the more important topics in advanced elementary algebra, such as quadratic equations, proportion and variation, true averages and graphs, and gives in addition a brief course in the essentials of plane trigonometry insofar as they apply to the solution of triangles. The latter part of the course is a miscellaneous mass of problems of varied character relating to almost every phase of undergraduate work in the college, special stress being laid upon farm management, dairy production, forest valuation and managment and advanced mensuration and survey problems. Those students who come from high school with a full three years in mathematics are not required to take this course or any further mathematics.

In the College of Agriculture we used to require of all boys entering the college with only two years of high school mathematics a term of Part I of higher algebra followed by a term of plane trigonometry. All freshman boys, regardless of the amount of high school requirements in mathematics, were required to take this course in plane trigonometry unless by chance they had also taken the plane trigonometry in the high school. The crowded condition of the curriculum in the college forced cutting down the time given to mathematics, resulting in the requirements above outlined. I believe the present arrangement is a mistake. We would get far better results with our students in the College of Agriculture if we could require at least three years of high school mathematics previous to entering college and still require *all* freshmen boys in the college to take this special course in elementary applied mathematics.

SCIENTIFIC INVESTIGATIONS RELATING TO AGRICULTURE

The preparation for this work, as in the case of the preceding phases, is a college course in agriculture and has to do more particularly with study along the line of special agricultural sciences, such as plant breeding and pathology, agricultural chemistry and soils, agricultural economics and rural sociology. The undergraduate work in these is largely of a preparatory character in general science and agriculture. The advanced work calls also for a considerable knowledge of physics and advanced mathematics not provided for in the curriculum of the College of Agriculture. To meet this need the mathematics and physics courses of our academic college are open as general electives for both undergraduate and graduate students wherever such may be needed.

Particular essentials in the various branches of mathematics. Let us now consider for a time in detail what should be taken up in the study of the various fields of elementary mathematics.

(a) *Arithmetic* has been pretty thoroughly discussed in the foregoing so that no further time will be given to it.

(b) *Algebra.* First fundamental principles. We must consider, of course, positive and negative numbers and the four fundamental operations, laying stress on the special products represented by the following type forms: $(a \pm b)^2$, $(a + b)(a - b)$, $(a \pm b)^3$; and the product of two binomials having a common term, $(a + b)(a + c)$.

These should be followed closely by the simpler factorable types as follows:

(1) Polynomials, each term of which contains a single monomial factor.

(2) The grouping type, such as: $am + bn + bm + an$.

(3) The perfect square of the form: $a^2 \pm 2ab + b^2$.

(4) The difference of two squares of the type form $a^2 - b^2$.

(5) The quadratic trinomial $a^2 + ax + b$.

The last three at least should be checked by visual geometric representation and demonstration.

(6) The general quadratic trinomial such as $ax^2 + bx + c$.

(7) The sum or the difference of the same powers of two quantites as $x^n \pm y^n$.

I believe the more intricate forms of factoring types should be omitted from high school courses. Then should come the algebraic concept of the fraction defined as merely an indicated, unperformed division, its manipulation therefore dependent on the more elementary laws of division. Then we should take up both simple and quadratic equations, solved by analytical methods. Lay little stress on solution of quadratics by formula which is liable to be made merely a matter of memory and so trip the student at a critical time by a lapse of memory. Carry the solution of simple equations into simultaneous equations of two or more unknown quantities. I do not, however, believe in using up the average student's time in long discussion of types of simultaneous quadratic equations. Bring in the elements of graphs just after the elements of quadratic equations and teach the student not only the graphical solution of the equation but also the depicting of statistical and other scientific data by the process of the mathematical graph, based on two rectangular axes. This is not difficult and if presented in the right manner can be covered in a week's time in ordinary high school work. We hear a great deal of talk from time to time about proportion in algebra as obsolete. The further the advanced student goes the more he uses the algebraic principle of proportion and variation. Hence, the high school teacher of the present day would do well to forget the suggestion that the teaching of proportion as such in algebra is obsolete. If it is ommitted in the high school work, we simply have to teach it in the college. Progressions should also be taught along with variation which is essential in almost every phase of college agricultural work. I do not believe in spending a great deal of time on algebraic demonstrations of rules. That is, I think so-called higher algebra for high school students is largely a waste of time. I would give the time usually allowed for such work to an extra half year of advanced elementary algebra. Some of the best of our high schools are doing this and have been for years past with good results. If it is found impossible to cover the usual course in elementary algebra thoroughly in the one year allowed provision for this extra half year of advanced work will take care of the shortage in the freshman year and also give the student the additional topics of progression, proportion, and variation, graphs, and more extensive problem analysis, by practice rather than by abstract proofs.

(c) *Geometry*. For the agriculture school student or the high school student preparing for the business of farming, the something like 270 or 280 principal propositions ordinarily covered in our standard texts on plane and solid geometry can by careful and systematic culling be cut from one third to one half and this should be done, due to the lack of room in the crowded curriculum. If it is done the course in plane geometry can quite tolerably well be covered in one six months' year of twenty-four weeks, five recitations per week with at least one month to spare for covering the essentials of solid geometry insofar as these apply to formulas for volumes of simple geometric solids. In the ordinary high school course of nine months, five times a week, it is customary to give one year to plane geometry, and the next half to solid geometry. I think this may well be put all into one year for agricultural students. It can be done. In handling the subject of geometry, the essentials are, of course, a clear conception of the geometrical elements, point, line and surface, as for example, define a line not merely as that which has length only but as the path of a moving point, or define a straight line, not merely as a line which does not change its direction throughout the entire course, but also as a portion of a circumference of a circle of infinite radius. That is, I would give a little more time to the elementary fundamental ideas and broaden the student's view. Then follow with perpendicular and parallel relations. These are few and definite and must be thoroughly mastered. Then comes the subject of triangles and other plane figures which can be made very brief and pointed, especial emphasis being given to the nature of the right triangle and its relation to the isosceles and equilateral triangle. We must have complete understanding of the principles of equality of plane figures, similarity of plane figures and areas of same, but let us cut out most of the mass, given in standard texts, of special original problems that lack usefulness and stick to the more practical phases. I do not mean by that that we should get away from insisting on a clear understanding and thorough memorizing of fundamental definitions, axioms and statements of theorems. That we must have. Do not insist on demonstration by the book but encourage originality and the simplest possible proofs of theorems. In solid geometry, practically all that is necessary is the elementary discussion of the plane and its relation to the line and the point, the perpendicular and parallel relations as in the case of plane figures, equality and similarity of solids, and the developments of formulas for the volumes of the simple solids. For those fundamental theorems which require more abstract reasoning but are absolutely essential to the development of volume formulas, physical demonstration apart from purely mathematical deduction is to my mind perfectly allowable and often helpful for the slow student. For example, he will probably have little difficulty in understanding the prism or the cylinder and mastering the fact that two prisms or cylinders, having equal bases, are in the same ratio as their altitudes, but when he gets to the pyramid or the cone, he may have more difficulty, for example, in proving the theorem that two pyramids having equivalent bases and equal altitudes are equivalent. I have seen the proof of this proposition absolutely stump some of my best students. If the teacher can secure two such pyramids of wood or other material of entirely different shape but being sure that their bases are equivalent and their altitudes equal and immerse each of these in turn in a measuring jar of water, even the dullest student can see for himself that they displace equal amounts of water. It is also possible to build up models with strips of xylonite, wire and thread which will ilustrate all sorts of relations between lines and planes and I think this should be done where a student or group of students are slow to grasp the analytical methods.

(d) *Plane Trigonometry.* I do not advocate the teaching of plane trigonometry in the high school. The opinions of many eminent professors of mathematics in our colleges to the contrary notwithstanding, I maintain that it is not a high school subject. Nevertheless, if it must be taught in the high schools, let us stick to the bare elements and make it purely an extension of plane geometry. If we teach trigonometry, we must necessarily teach the elements of logarithms which means the use of the common logarithm table and if we are to teach logarithms, let us teach it in our elementary and advanced algebra along with the subject of exponents, showing that a logarithm is what it is, merely a special type of exponent. Do not try to explain how it is arrived at but merely how it is made up and what its action is in the ordinary operations of multiplication, division, involution, and evolution. Then in trigonometry, let the teacher pass, by as clear explanations as possible, from the similarity of plane triangles whose sides are proportional to the fundamental idea and definition of the trigonometric functions. There is no need to spend any great amount of time in the study of the inter-relations between the functions apart from the reciprocal relations which appear in the primary definitions. It becomes at once apparent to the interested student that these fundamentl definitions are the laws for the solution of right triangles and it is quite possible to give all the necessary laws for the solution of oblique triangles in five simple geometrical demonstrations covering about as many pages in an ordinary text. There is no need to drag the student through fifty or sixty pages of long analytical discussion, leading him up by slow stages to the proof of the tangent law if he has thoroughly committed the definitions of the trigonometric functions and understands what they represent. Extensive solution of simple problems in triangles and other simple plane figures will fix the laws of trigonometry very firmly in the mind of the student and at the same time arouse his interest.

General Considerations. Let us be firm and concise on fundamental definitions, *e. g.*, the pupil must be clear in his disinction between a factor and a term but let us banish also difficult scientific names. If two triangles are equal under certain conditions, let them be equal, not congruent. Let us pick the shortest proof we can for difficult propostions, not the longest and most complicated ones. For example, in the comparison of plane figures and solids, having one or two similiar dimensions, let us not insist that the student must drag in, and without understanding, the abstract theorem of limits. Let us look for a moment at the theorem. "Two rectangles having equal altitudes are to each other as their bases." Assume Case I, the commensurable case, to have been proven. Case 2 is where the bases are incommensurable. Let us proceed as follows: first the divisor in the first figure, being established, it is always greater than the remainder in the second figure. Second, now continue to reduce the divisor of the first figure; the remainder in the second figure will likewise be reduced. Third, if we continue to reduce the divisor until it has become the smallest possible quantity, the remainder will then have become less than the smallest possible quantity or zero; the two figures are then commensurable and the theorem is proven by Case 1.

Conclusion: I have dealt largely in generalization. It has been my intention to show that the requirements in mathematics for agricultural study are not extensive but intensive, not complex and advanced but simple and elementary, not theoretical and abstract, but practical and concrete. It is in education today as in other things; the interesting and new theory and unexplored realm of knowledge, the intellectual luxury, if you please, of yesterday becomes the commonplace requirement of mental equipmnt requisite for success today. The result is our school and college curricula are crowded with essential requirements in many lines and the cry is constantly for the stronger and plainer, more utilitarian lines of mental architecture and the dispensing with the purely ornamental furbelows and friezes of the past to make room for the many essentials. We mathematicians, ever conscious of the lasting worth and fundamental strength of the science, still must hear and strive to heed the demand of the hour if we would not see the mathematics temporarily lose—for it cannot lose it permanently—the place of power and strength it has so long and rightly held in the educational structure.